Question 1:

Find all horizontal, vertical asymptotes and oblique asymptotes if any:

1. $f(x)=\dfrac{x^4}{x^2+4x+4}$
2. $f(x)=\dfrac{x^2-4}{x^2+4}$
3. $f(x)=\dfrac{x^2-2x-3}{x^3-x}$
4. $f(x)=\dfrac{x^3-2x}{x^2-x}$

Question 2:

For the functions $f(x)=\dfrac{4x}{3x-6}$, and $f(x)=6x(x-1)^3$ find:

a. the domain

b. the x-intercept(s) and the y-intercept

c. the vertical asymptote(s) and the horizontal asymptote(s) (if any)

d. The increasing, decreasing intervals and all local max and Min if any

e. The concave upward and downward intervals and all inflection point(s) if any

f. Graph the function

Question 3: Sketch the graph of a function with the following properties:

1. Domain: $(-\infty,0)\cup(0,\infty)$
2. Vertical asymptote: $x=0$ and No Horizontal asymptote
3. $f(-3)=-3$ *and* $f(3)=3$.
4. $f'(-3)=f'(3)=0$
5. $f'(x)>0$ on $(-\infty,-3)\cup(3,\infty)$ and $f'(x)<0$ *on* $(-3,0)\cup(0,3)$.
6. $f''(x)>0$ on $(0,\infty)$ and $f''(x)<0$ *on* $(-\infty,0)$.

Question 3:

Find the absolute max and min, if either exists, for each function.

1. $f(x) = x^2 - 2x + 3$ on $[0,3]$
2. $f(x) = x + \frac{4}{x}$ on $(0,4]$
3. $f(x) = (x-5)^5 + 1$ on [3,6]

Question 4:

Find the derivative of the following Functions

1. $f(x) = 4x + e^{-2x} + \ln(x^2 + 1)$
2. $f(x) = \frac{1}{x} + \log_3(2x^2 + 3x - 1)$
3. $f(t) = e^{(t^2 + 4)}$
4. $f(x) = 3\cos(x^2 - 2x)$
5. $f(x) = \tan(\frac{x^3 - 3x}{x^2})$

Question 5:

Use Riemann sum to find the approximate value of the following definite integrals and check you answer using the fundamental theorem of calculus:

1. $\int_1^3 1 - 2x^2 \; dx$
2. $\int_0^2 x^3 \; dx$

Question 6: Find the following integrals:

1. $\int (3x^2 - \frac{2}{x^2}) dx$
2. $\int \frac{e^x - 3x^2}{2} dx$

3. $\int_1^3 (\sqrt[3]{x^2} - \frac{4}{x^3})$

4. $\int \frac{x^2+2x-1}{x} dx$

5. $\int_0^1 \frac{4x^3}{\sqrt{x^4+3}} dx$

6. $\int_2^4 \frac{x-1}{(x^2-2x+3)^3} dx$

Question 7: Optimization Problems

PROBLEM 1:
A window is being built and the bottom is a rectangle and the top is a semicircle. If there is 12 meters of framing materials what must the dimensions of the window be to let in the most light?

PROBLEM 2: Determine the point(s) on $y = x^2 + 1$ that are closest to $(0, 2)$

PROBLEM 3: We need to enclose a field with a fence. We have 500 feet of fencing material and a building is on one side of the field. Determine the dimensions of the field that will enclose the largest area.

PROBLEM 4: Determine the area of the largest rectangle that can be inscribed in a circle of radius 4.